



FIGURE 12. MAXIMUM PRESSURE-TO-STRENGTH RATIO,  $p/\sigma_1$ , IN MULTI-RING CONTAINER WITH HIGH-STRENGTH LINER BASED ON THE FATIGUE TENSILE STRENGTH OF LINER

The  $k_n$ ,  $n \geq 2$  in Equation (50) are equal as shown by Equation (48). Whereas,  $p/\sigma_1$  depended only upon  $\alpha_r$  and  $K$  (Equation (47)),  $p/\sigma$  depends on  $N$ ,  $k_n$ , and  $\alpha_m$  in addition.

The ratio  $p/\sigma$  can also be limited by the requirement on Relations (9) and (12) that the mean shear stress  $S_m$  in cylinder No. 2 at  $r_1$  obeys the relation  $S_m \geq 0$ .  $S_m = 0$  gives

$$\left(\frac{p}{\sigma}\right)_{\text{limit}} = \frac{2}{3} \frac{(K^2 - 1)}{K^2} k_1^2 \quad (51)$$

The limit curves are plotted in Figure 13. As evident from Figure 13, the pressure limit for the outer rings can be increased by increasing  $k_1$ . This means that the liner has a great effect on  $p$ . The strength of the liner,  $\sigma_1$ , influences  $p$  in Equation (47). The size of the liner,  $k_1$ , limits  $p$  in Equation (51).

Whether or not  $p/\sigma$  can be allowed as high as the limit, however, depends on the other factors  $N$ ,  $\alpha_r$ ,  $K$ , etc., as shown by Equation (50). This dependence is rather complicated. Example curves of  $p/\sigma$  are plotted in Figures 14 and 15 for  $\alpha_r = 0.5$  and  $\alpha_m = -0.5$ . As shown by these curves  $p/\sigma$  increases with  $N$  and also increases with  $k_1$  for  $N = 5$ ,  $K \geq 6.5$ .

Suppose  $p = 300,000$  psi as determined from Equation (47) for  $\alpha_r = 0.5$  and  $\sigma_1 = 300,000$  psi. Then from Figure 15,  $K$  must be 9.0 for  $k_1 = 1.75$  and  $N = 5$  if  $\sigma = 210,000$ . Thus, the multi-ring cylinder must be quite large in size to support maximum repeated pressures.

The interferences  $\Delta_n$  and residual pressures  $q_n$  have yet to be determined for the multi-ring container. Since the liner and the outer rings are assumed to be made from two different materials, thermal expansions must be included in the interference calculations. It is assumed that no thermal gradients exist; all components reach the same temperatures uniformly. Therefore, the interference required between the liner and the second cylinder is expressed as

$$\frac{\Delta_1}{r_1} = -\frac{u_1(r_1)}{r_1} + \frac{u_2(r_1)}{r_1} - \alpha_1 \Delta T + \alpha_2 \Delta T \quad (52)$$

where

$\Delta_1$  = manufactured interference

$u_1(r_1)$  = radial deformation of liner at  $r_1$  due to residual pressure  $q_1$  at  $r_1$

$u_2(r_1)$  = radial deformation of cylinder No. 2 at  $r_1$  due to residual pressures  $q_1$  at  $r_1$  and  $q_2$  at  $r_2$

$\alpha$  = coefficient of thermal expansion at temperature

$\Delta T$  = temperature change from room temperature.